On the origins and foundations of Laplacian determinism

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Laplace (1814):

> “An intelligence which, for one given instant, would know all the forces by which nature is animated and the respective situation of the entities which compose it, if besides it were sufficiently vast to submit all these data to mathematical analysis, would encompass in the same formula the movements of the largest bodies in the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.”

Claims:

- Laplace's statement does not necessarily have to be interpreted as stating a fact about physics.
- The statement has a strong background in eighteenth century 'Leibnizian' metaphysics; it is ultimately based on a re-interpretation of Leibniz' principle of sufficient reason and law of continuity.

1. PHYSICAL FOUNDATIONS

The common interpretation of Laplace's statement:

> For each system in classical mechanics, there are equations of motion of the form $\frac{d^2r}{dt^2} = F(r)$, which have a unique solution for given initial conditions $r(t_0) = r_0$ and $\frac{dr}{dt}(t_0) = v_0$.

Problems with this interpretation:

(1) Laplace never says this explicitly.

- The context of his statement is not mechanics but probability theory.
- In 1773: "…if we conceive of an intelligence which, at a given instant, encompasses all the relations between the beings of this universe, it may determine, for a certain time in the past or the future, the respective positions, the motions, and generally the conditions of all those beings" – this was in a similar context.

1. PHYSICAL FOUNDATIONS

(2) Laplace had no proof.

- Existence and uniqueness theorems for differential equations were only developed later on: Cauchy, 1820s; Lipschitz, 1876.

  Lipschitz:

  $\frac{d^2r}{dt^2} = F(r)$ has a unique solution iff there is a $K > 0$ such that for all r_1, r_2 : $|F(r_1) - F(r_2)| \leq K|r_1 - r_2|$

- Indeterminism is possible in cases which do not satisfy Lipschitz' requirement for uniqueness of solutions (Poisson, 1806; Boussinesq, 1878; Norton, 2003).
- Laplace was aware that differential equations could have non-unique solutions, he even published on this issue (1772).
- It may have been clear to Laplace that the equations he worked with in his physics always had a unique solution. This is quite reasonable in celestial mechanics , but less clear in other areas of physics.

2. LAPLACE'S ARGUMENT

Laplace's statement in full:

"Current events are connected with preceding ones by a tie based upon the evident principle that a thing cannot come to existence without a cause which produces it. This axiom, known as the *principle of sufficient reason*, extends even to the most indifferent acts. The most free will cannot give rise to these indifferent acts without a determinative motive; for if, in two cases with exactly similar circumstances, the will acted in the one and refrained from acting in the other, its choice would be an effect without a cause. It would then be, says Leibniz, the blind chance of the Epicureans. The contrary opinion is an illusion of the mind which, losing sight of the evasive reasons of the choice of the will among indifferent things, convinces itself that it has determined itself independently and without motives.

We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. An intelligence which, for one given instant, would know all the forces by which nature is animated and the respective situation of the entities which compose it, if besides it were sufficiently vast to submit all these data to mathematical analysis, would encompass in the same formula the movements of the largest bodies in the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes."

3. DETERMINISM BEFORE LAPLACE

‘Laplacian’ determinism in the 18th century:

- Maupertuis (1756): Sur la divination.
- Boscovich (1758): Theoria philosophiae naturalis.
- Condorcet (1768): Essais d'analyse: Lettre dite du marquis de Condorcet à M. d'Alembert, sur le système du monde et le calcul intégral.
- D'Holbach (1770): Système de la nature ou des lois du monde physique et du monde moral.
- Diderot: undated fragment in his archives.

3. DETERMINISM BEFORE LAPLACE

D'Holbach (1770):

Even in a storm or in a whirlwind of dust, no molecule moves by chance.

> "A geometrician, who would exactly know the different forces which act in these two cases, and the properties of the molecules that are moved, could demonstrate, that, according to given causes, each molecule acts precisely as it ought to act, and could not have acted otherwise than it does."

Similarly,

> "In the terrible convulsions which sometimes agitate political societies, and which often produce the overthrow of an empire, there is not a single action, a single word, a single thought, a single desire, a single passion in the agents which participate in the revolution as destructors or as victims, which is not necessary, which does not act as it ought to act, which does not infallibly produce the effects that it has to produce, according to the place which these agents occupy in this moral turmoil. This would seem evident for an intelligence that would be in a position to know and appreciate all the actions and reactions of the minds and the bodies of those who contribute to this revolution."

3. DETERMINISM BEFORE LAPLACE

Leibniz (1702):

> “There is no doubt that a man could make a machine which was capable of walking around a town for a time, and of turning precisely at the corners of certain streets. And an incomparably more perfect, although still limited, mind could foresee and avoid an incomparably greater number of obstacles. And this being so, if this world were, as some think it is, only a combination of a finite number of atoms which interact in accordance with mechanical laws, it is certain that a finite mind could be sufficiently exalted as to understand and predict with certainty everything that will happen in a given period. This mind could then not only make a ship capable of getting itself to a certain port, by first giving it the route, the direction, and the requisite equipment, but it could also build a body capable of simulating a man.”

4. THE LAW OF CONTINUITY

Condorcet (1768):

> "…if the law of continuity is not violated in the universe, one could regard its state at every instant as the result of what had to happen to matter once arranged in a certain order and then abandoned to itself. An Intelligence that would then know the state of all phenomena at a given instant, the laws to which matter is subjected, and their effects after a certain period of time, would have perfect knowledge of the *System of the World*."

4. THE LAW OF CONTINUITY

The “law of continuity” in the *Encyclopédie*:

> “Continuity, (law of) is a principle that we owe to M. Leibniz, and which teaches us that nature takes no leaps, and that a being never passes from one state into another, without passing through all the different states that one can conceive in between them. This law follows, according to M. Leibniz, from the axiom of sufficient reason.” (Formey/ d'Alembert, 1754)

The law of continuity is a central principle in Leibniz’ thought.

- all transitions are gradual.
- there is continuity in the transition to a limit.

4. THE LAW OF CONTINUITY

Encyclopédie, continued:

> “For each state in which a being finds itself, there must be a sufficient reason why this being finds itself in this state rather than in any other; and this reason can only be found in the antecedent state. This antecedent state thus contained something which has given rise to the current state which has followed it; so that these two states are connected in such a way that it is impossible to put another one in between; because if there were a possible state between the current state and that which immediately preceded it, nature would have left the first state without already being determined by the second to abandon the first; there would thus not have been a sufficient reason why it would have passed to this state rather than to any other possible state. Thus no being passes from one state to another, without passing through the intermediary states; just as one does not go from one city to another without going over the road between them.” (Formey/ D'Alembert, 1754).

CONCLUDING REMARKS

- Laplace's determinism is based on the idea that states of a physical system are continuously determined by previous states. Motivation: principle of sufficient reason, law of continuity.
- This idea of continuous determination also lies at the basis of the application of differential calculus in physics.
- Therefore, Laplace's determinism and the idea that for each system in mechanics there are equations with unique solutions have a *common basis*.