Is a gas equal to a collection of molecules? On the modal logic of reduction

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Thermodynamics and statistical mechanics

- Thermodynamics:
  - concerns macroscopic systems (e.g., a gas)
  - systems described in terms of few degrees of freedom
  - time-irreversible dynamics (Second Law)
- Statistical mechanics:
  - based on micro-constituents of systems
  - probabilistic theory to handle vast number of degrees of freedom
  - underlying dynamics is time-reversible

Clearly, any thermodynamic system **is** (also) a micro-system

- minimally: "is constituted by"
- what about: "is identical with"?

How to understand the interrelation?

This talk: focus on the modal logic of identity

Disclaimer

This talk is not primarily about science or metaphysics, but about a logical framework that allows to discuss different views.

A useful logic should be neutral w.r.t. science and metaphysics. It should have the resources to represent different scientific or metaphysical viewpoints.

Main claim:

Case-intensional first-order logic (CIFOL) offers a useful perspective on the interrelation of thermodynamics and statistical mechanics.

Analogy: constitution of a biological individual by its matter

Overview

Case-intensional first-order logic

Tracing individuals

Thermodynamics

Quantified temporal/modal logic: standard picture

How to build logical models for individual things, their identity and what is possible for them?

Quantified modal logic:

- Many different systems
- Typical: worlds and world-relative domains
- Further aspects often driven by metaphysical considerations
- Standard basic notions:
  - Set of worlds W
  - Domain of individuals D_w for $w \in W$
- Key issues: trans-world identity? Variables as rigid designators? Counterpart theory?

Need for a well-constructed "substance-language" (Prior)

Case-intensional logic: Bressan

The most persistently overlooked important contribution to quantified modal logic:

Aldo Bressan, *A general interpreted modal calculus*, New Haven, CT: Yale University Press 1972.

Current developments: Case-intensional first order logic

Nuel Belnap & Thomas Müller (2013), "CIFOL: Case-intensional first order logic. (I) Toward a theory of sorts", *J. Phil. Logic*, online first, DOI 10.1007/s10992-012-9267-x.

Nuel Belnap & Thomas Müller (2013), "BH-CIFOL: Case-intensional first order logic. (II) Branching histories", *J. Phil. Logic*, forthcoming.

CIFOL semantics (i)

- Cases $\gamma \in \Gamma$; extensional domain D
- Individual term α (constant, variable, ...) has
  - extension in each case: $ext_\gamma(\alpha) \in D$
  - intension: pattern of extensions, $int(\alpha) \in (\Gamma \mapsto D)$
- Assignment δ: $Var \mapsto (\Gamma \mapsto D)$ (intensional variables)
- General link extensions / intension for expressions ξ:

$$ext_{\gamma,\delta}(\xi) = (int_\delta(\xi))(\gamma); \qquad int_\delta(\xi) = \lambda\gamma(ext_{\gamma,\delta}(\xi))$$

- Predication is intensional:
  - Standard conception: extensional predication, $int(P) \in \Gamma \mapsto (D \mapsto \mathbf{2})$
  - Here: intensional predication, $int(P) \in \Gamma \mapsto ((\Gamma \mapsto D) \mapsto \mathbf{2})$. Uniform clause:

$$ext_{\gamma,\delta}(P\alpha) = (ext_{\gamma,\delta}P)(int_\delta\alpha) \in \mathbf{2}$$

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CIFOL semantics (ii): intensional predication

Four cases, $\gamma_1, \ldots, \gamma_4$; domain $D = \{$a,b,c,d,e,f,g,h,j,k,l,m,n,*$\}$;
term "Jack" with intension jkln

Property \ Case	γ_1	γ_2	γ_3	γ_4
Horse	abcd ef** jkln	abcd ef** jkln	abcd ef** jkln	abcd ef** jkln
Black	a--- e---	-b-- -f--	--c-	---d
Brown	j---	-k--	--l-	---n
Eating	a--- j---	-b-- -f--	$\varnothing$	---n

$\gamma_1 \models$ Eating(Jack), i.e., ext_{γ_1}(Eating(Jack)) $= T$
$\gamma_2 \models \neg$Eating(Jack), i.e., ext_{γ_2}(Eating(Jack)) $= F$

CIFOL semantics (iii)

- Alethic modality: simple **S5**:
  $\gamma, \delta \models \Box\phi$ iff for all $\gamma' \in \Gamma$: $\gamma', \delta \models \phi$
- Quantification: variables for individual intensions:
  $\gamma, \delta \models \forall x \phi$ iff for all $\bar{z} \in (\Gamma \mapsto D)$: $\gamma, \delta[\bar{z}/x] \models \phi$
  $\Rightarrow$ BF and CBF are valid
  N.B.: Can't read "$\forall x$" as "for all things x"

- Identity is extensional:
  $\gamma, \delta \models \alpha = \beta$ iff $ext_{\gamma,\delta}\alpha = ext_{\gamma,\delta}\beta$
- Only necessary identity $\Box\alpha = \beta$ allows replacement
- Existence $Ex \Leftrightarrow_{df} x \neq *$ via "throwaway" $* \in D$
  E.g., $1960 \models \text{Thomas} = *$

- Easy handling of λ-predicates/-predications, λ-operators/-terms, definite descriptions

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CIFOL semantics (iv): Summary

CIFOL offers expressivity:

- All terms have intensions (and an extension in each case)
  ⇒ represent concrete individuals via individual intensions;
  extra-logical issue which individual intensions are things
  and what the extensions are (not themselves things)
- Predication is intensional
  ⇒ have handle for sortal / dispositional properties
- Identity is extensional
  ⇒ fine-grained treatment of identity provides
  neutral platform for discussing constitution/overlap
  handle for new discussion of reduction

No tweaking of the logic for the sake of metaphysics.

The interface to metaphysics, science etc. comes via powerful definitions: defined properties of properties.

CIFOL's interface for tracing

- Interface to metaphysical/scientific discussion via *definable properties of properties*, not via rigid designators

- EXT: P is *extensional* $\Leftrightarrow_{df} \Box\forall x \forall y\,(x = y \rightarrow (Px \leftrightarrow Py))$
- MC: P is *modally constant* $\Leftrightarrow_{df}$ $\forall x\,(\Diamond Px \rightarrow \Box Px)$
- MS: P is *modally separated* $\Leftrightarrow_{df}$ $\Box\forall x \forall y\,(Px \wedge Py \rightarrow (\Diamond(x \neq * \wedge x = y) \rightarrow \Box x = y))$
- ABS: P is an *absolute property* $\Leftrightarrow_{df}$ P is MC and MS

- Every ABS property has extensional companions: $P^e x \Leftrightarrow_{df} \exists y\,(Py \wedge x = y)$; $P^{e!} x \Leftrightarrow_{df} P^e x \wedge x \neq *$

Slogan: Sortal properties are absolute

The power of absolute properties

Modal separation explains how absolute properties do two jobs:

- Extensional classification
- Tracing of individuals across cases

Generally: Individuals falling under an absolute property can be captured by a single case. If $ABS(P)$, then

$$\Box[P^{e!}\alpha \rightarrow \exists x\,(x = \alpha \wedge Px \wedge \forall y((Py \wedge y = \alpha) \rightarrow \Box x = y))]$$

Slogan: Individual substances (concrete things) fall under natural absolute properties

Logical features

$a = b \wedge \neg\Box a = b$

- easy examples via a name and a description: "Obama = the president", but not necessarily
- CIFOL also allows two substance terms. Let l be the lump of matter constituting Tibbles the cat at time t: "Tibbles = l" is true at time t, but at no other time

$a = b \wedge \Diamond Fa \wedge \neg\Diamond Fb$

- what's possible for the cat's matter needn't be possible for the cat

Tracing individuals: "Follow the man"

- "Thomas (τ) is the speaker (σ)": only extensional identity
  $\tau = \sigma$; $\neg\Box(\tau = \sigma)$
- "The speaker is a man"
  $Man(\tau)$; $\neg Man(\sigma)$; $Man^e(\sigma)$
  "Thomas" is a name *for* a man;
  "the speaker" is only a name *of* a man
- Talking about traits:

  "Thomas is a non-smoker"
  $\Box\neg Smokes(\tau)$ (better: in most past cases)

  "The speaker is a non-smoker"
  Need to trace the individual man from case to case:
  $\exists x\,[Man(x) \wedge x = \sigma \wedge \Box(\neg Smokes(x))]$
  (won't help to trace the speaker)

Thermodynamics

"The gas (thermodyn. system) is a collection of molecules."

True enough. Two natural sortals, T and C. We have

$$\Box\forall x[(Tx \wedge Ex) \rightarrow \exists y[Cy \wedge x = y]]$$

Persistence: Assume (for simplicity) indestructible molecules. What about the gas? Can cease to exist, in various ways

$$\Box\forall x[Cx \rightarrow \Box Ex] \quad \wedge \quad \Diamond\exists x((Tx \wedge Ex) \wedge \Diamond\neg Ex)$$

Thus, the identity of a thermodynamic system and a collection of molecules is generally contingent:

$$\neg\Box\forall x[(Tx \wedge Ex) \rightarrow \exists y[Cy \wedge \Box x = y]]$$

Thermodynamics and the Second Law

"A thermodynamic system is a collection of molecules."

The Second Law holds for thermodynamic systems, e.g., gases. But what about collections of molecules?

Sx: The entropy of S increases over time

$$\Diamond\exists x[Tx \wedge Ex \wedge Sx \wedge \Box(Ex \rightarrow Sx) \wedge \exists y[Cy \wedge x = y \wedge Sy \wedge \Diamond\neg Sy]]$$

Idea: You can motion-invert the collection of molecules that *is* the gas in the case in point, but then the entropy of the collection will decrease, and thus *the collection is not a thermodynamic system any more*. The thermodynamic system has ceased to exist.

Some fine print

- CIFOL just provides a framework. The rest of the discussion is for science and metaphysics – but the framework is available.
- What about the extensions? *No claim needs to be made* about them. Use microstates if you like.
- "Agnostic non-reductive monism"
- Short-term fluctuations?
  Distinguish idealization in identifying thermodynamic systems from violations of Second Law.
- Role of Past Hypothesis or similar principle?
  Explanation of why it's so easy to find thermodynamic systems in nature, while it's so hard to find collections of molecules that aren't thermodynamic systems.

Thanks for your attention!