On the Significance of the Gottesman-Knill Theorem

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What is quantum computing?

Uses the resources of quantum mechanics to solve computational problems faster and more efficiently than is possible using (known) classical means.

In some cases, this 'quantum speedup' can be dramatic (exponential).

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- Shor's algorithm: a provably efficient quantum algorithm for factoring.
  - Makes the factoring problem tractable.

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Quantum Entanglement:

- Exponentially larger state space efficiently accessible to quantum (as opposed to classical) systems.
- Can be used as a resource to enable quantum speedup.

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  - Is a pure entangled state sufficient to enable speedup?
  - Gottesman-Knill theorem: a counter-example?

Outline

1. Introduction.
2. The Gottesman-Knill theorem.
3. Analysis of the Gottesman-Knill operations.
    i. The CHSH inequality.
    ii. The GHZ argument.
4. Conclusion.

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The Gottesman-Knill theorem

Any algorithm using only these is efficiently simulable classically:

- State preparation of product states of qubits in the computational (i.e., $\{|0\rangle, |1\rangle\}$) basis
- Measurements of (products of) Pauli observables.
- "Clifford group" gates:
  - Pauli gates
  - Phase gates
  - CNOT gates
  - Hadamard gates
- Clifford group gates conditioned on classical bits

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Recall that the significance of entanglement for pure-state computations is derived from the fact that unentangled pure states ... of n qubits have a description involving poly(n) parameters (in contrast to $O(2^n)$ parameters for a general pure state). But this special property of unentangled states (of having a 'small' descriptions [sic.]) is contingent on a particular mathematical description, as amplitudes in the computational basis. If we were to adopt some other choice of mathematical description for quantum states (and their evolution), then, although it will be mathematically equivalent to the amplitude description, there will be a different class of states which will now have a polynomially sized description; ...

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With this in mind we see that the significance of entanglement as a resource for quantum computation is not an intrinsic property of quantum physics itself, but is tied to a particular additional (arbitrary) choice of mathematical formalism for the theory. ... An explicit example of an alternative formalism and its implications for the power of quantum computation is provided by the so-called stabilizer formalism and the Gottesman-Knill theorem ... (Jozsa & Linden, 2003, 2029-2030).

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$$\frac{1}{2^{n/2}}(|0\rangle + |1\rangle)_1(|0\rangle + |1\rangle)_2 \dots (|0\rangle + |1\rangle)_n \quad (1)$$

$$= \frac{1}{2^{n/2}}(|00\dots00\rangle + |00\dots01\rangle + \cdots + |11\dots10\rangle + |11\dots11\rangle. \quad (2)$$

· Eq. (1) is exponentially smaller than Eq. (2).

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- Is there a physical reason?

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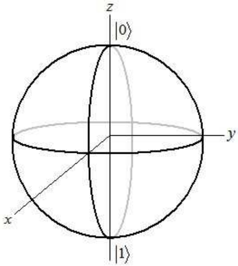
Combined effect of any sequence of only GK operations is equivalent to the measurement of one of the Pauli observables: $\pm X$, $\pm Y$, $\pm Z$ on either $|0\rangle$ or $|1\rangle$.

Pauli observables

X, Y, Z: π rotations of the Bloch sphere about the x, y, and z axes respectively.



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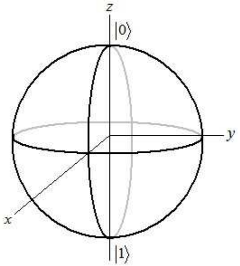

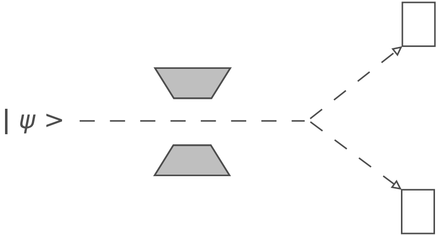

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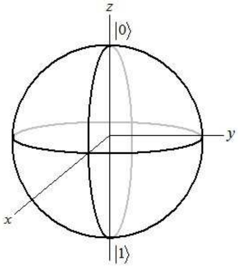
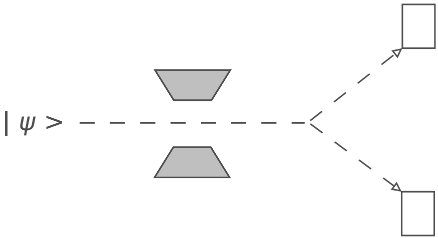
Precisely the orientations for which it is possible to provide a LHVT to reproduce the statistics associated with a Bell state.





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$$|\langle \sigma_m \otimes \sigma_n \rangle + \langle \sigma_m \otimes \sigma_{n'} \rangle| + |\langle \sigma_{m'} \otimes \sigma_n \rangle - \langle \sigma_{m'} \otimes \sigma_{n'} \rangle| \leq 2.$$

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But <u>no conflict</u> when $\hat{m}$ and $\hat{n}$, $\hat{m}$ and $\hat{n}'$, $\hat{m}'$ and $\hat{n}$, and $\hat{m}'$ and $\hat{n}'$ are all oriented at angles $\propto \pi/2$.

Physical motivation for the GK theorem

- Measurement statistics on systems subjected to only GK operations are recoverable by a LHVT.
- But joint experiments in an LHVT can be described using product states, hence “compressible”.
- So unsurprising that these operations are efficiently simulable with a classical computer.

The GHZ argument

Cf. Greenberger, Horne, & Zeilenger (1989); Greenberger, Horne, Shimony, & Zeilenger (1990); Mermin (1990); Clifton, Redhead, & Butterfield (1991)

Demonstrates an incompatibility between the predictions of certain LHVTs and QM predictions.

- uses measurements of Pauli observables exclusively,

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Consider three spatially separated (in the same plane) spin-1/2 systems a, b, c are in the state:

$$|\mathrm{GHZ}\rangle_3 = \frac{1}{\sqrt{2}}(|0\rangle_a|0\rangle_b|0\rangle_c + |1\rangle_a|1\rangle_b|1\rangle_c).$$

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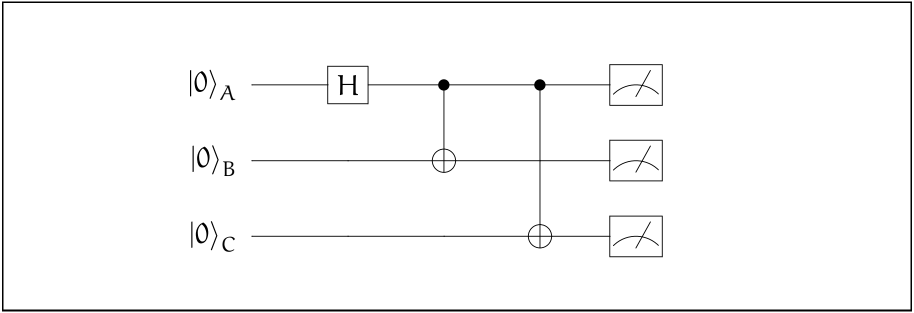
Point out that this contradicts quantum mechanical prediction of:

$$v(X^a \otimes X^b \otimes X^c) = 1.$$

Quantum circuit diagram representation of the GHZ argument



Implements:

$$\text{CNOT}_{13}\text{CNOT}_{12}\text{H}_1|0\rangle|0\rangle|0\rangle.$$

All of these (including subsequent X, Y measurements) are allowed GK operations.

Physical motivation for the GK theorem??

- Measurement statistics on systems subjected to only GK operations are recoverable by a LHVT??

HVT to recover GHZ_3 predictions (Tessier, 2004; Tessier et al., 2005)

	q_A	q_B	q_C
X	R_2R_3	R_2	R_3
Y	$iR_1R_2R_3$	iR_1R_2	iR_1R_3
Z	R_1	R_1	R_1
I	1	1	1

- q_A, q_B, and q_C: qubits for Alice, Bob, Chris.
- R_k: random variables; take a value of ± 1 with equal probability (interpreted epistemically).
- $R_k^2 = (\pm 1)^2 = 1$.

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Determining measurement outcomes:

- $v(XXX) = R_2R_3R_2R_3 = 1$
- $v(XYY) = R_2R_3iR_1R_2iR_1R_3 = i^2 = -1$
- $v(YYX) = iR_1R_2R_3iR_1R_2R_3 = i^2 = -1$
- $v(XYI) = R_2R_3iR_1R_2 = \pm i \Rightarrow \pm 1$
- etc.

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- $v(XYI) = R_2R_3iR_1R_2 = \pm i \Rightarrow \pm 1$
- etc.

Recovers all of the predictions of quantum mechanics for any product of Pauli experiments on the GHZ_3 state.

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Problem: outcomes are inconsistent with one another.

E.g.,

$$\begin{aligned} v(XYY) &= R_2R_3iR_1R_2iR_1R_3 = -1 \\ &\neq v(XII) \times v(IYI) \times v(IIY) = (R_2R_3)(R_1R_2)(R_1R_3) = 1 \end{aligned}$$

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- Bob agrees to send a single classical bit to Alice: 0 if Bob measured Y or 1 otherwise.
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- Also true for any circuit consisting exclusively of Gottesman-Knill operations (details given in Tessier 2004).
- Scheme no longer works for non-GK operations. Amount of classical communication conjectured to grow exponentially (Tessier, 2004).

Tessier (2004) on the significance of the GK theorem

Our results yield an alternative perspective on the GK theorem, and demonstrate that we may replace the nonlocal hidden variables represented by the stabilizer generators with LHVs and an amount of classical communication that scales efficiently with the size of the problem. This is a general feature of quantum circuits obeying the constraints of the GK theorem since, as our model illustrates, such circuits do not utilize the full capabilities of the available entanglement in the probability distributions that they generate (Tessier, 2004, 103).

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Y	iR_1R_2	iR_1R_2	iR_3
Z	R_1	R_1	1
I	1	1	1

$$|000\rangle \xrightarrow{H_A} \frac{(|0\rangle + |1\rangle)|00\rangle}{\sqrt{2}} \xrightarrow{CNOT_{AB}} \frac{(|00\rangle + |11\rangle)|0\rangle}{\sqrt{2}}$$

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Z	R_1	R_1	1
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$\xrightarrow{CNOT_{AC}}$

	q_A	q_B	q_C
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Y	$iR_1R_2R_3$	iR_1R_2	iR_1R_3
Z	R_1	R_1	R_1
I	1	1	1

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Outcomes completely determined by R_k

R_1	R_2	R_3	"XYY" $R_2R_3iR_1R_2iR_1R_3$	"YXY" $iR_1R_2R_3R_2iR_1R_3$	"YYX" $iR_1R_2R_3iR_1R_2R_3$	"XXX" $R_2R_3R_2R_3$
+1	+1	+1	−1	−1	−1	+1
+1	+1	−1	−1	−1	−1	+1
+1	−1	+1	−1	−1	−1	+1
+1	−1	−1	−1	−1	−1	+1
−1	+1	+1	−1	−1	−1	+1
−1	+1	−1	−1	−1	−1	+1
−1	−1	+1	−1	−1	−1	+1
−1	−1	−1	−1	−1	−1	+1

R_1	R_2	R_3	"XII" R_2R_3	"IYI" iR_1R_2	"IIY" iR_1R_3	...
+1	+1	+1	+1	+1	+1	
+1	+1	−1	−1	+1	−1	
+1	−1	+1	−1	−1	+1	
+1	−1	−1	+1	−1	−1	
−1	+1	+1	+1	−1	−1	
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−1	−1	+1	−1	+1	−1	
−1	−1	−1	+1	+1	+1	

Outcomes not completely determined by R_k

			"XYY"	"YXY"	"YYX"	"XXX"
R_1	R_2	R_3	$R_2R_3iR_1R_2iR_1R_3$	$iR_1R_2R_3R_2iR_1R_3$	$iR_1R_2R_3iR_1R_2R_3$	$R_2R_3R_2R_3$
$+1$	$+1$	$+1$	-1	-1	-1	$+1$
$+1$	$+1$	-1	-1	-1	-1	$+1$
$+1$	-1	$+1$	-1	-1	-1	$+1$
$+1$	-1	-1	-1	-1	-1	$+1$
-1	$+1$	$+1$	-1	-1	-1	$+1$
-1	$+1$	-1	-1	-1	-1	$+1$
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			"XII"	"IYI"	"IIY"	…
R_1	R_2	R_3	R_2R_3	iR_1R_2	iR_1R_3	
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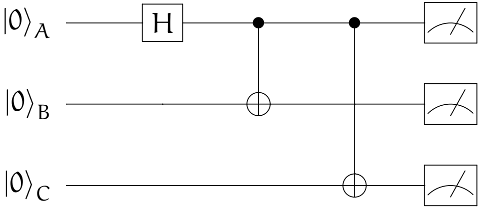
Is there a better, more illuminating, way to understand the significance of this model?

Received interpretation of the model



$R_2 R_3$	"XII"
$i R_1 R_2$	"IYI"
$i R_1 R_3$	"IIY"

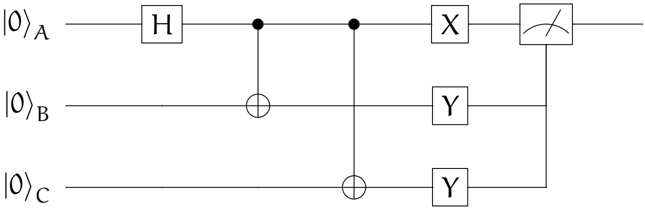
Interfering i terms represent a nonlocal influence.

- Yield contextual measurement outcomes:

$$\begin{aligned} \nu(XYY) &= R_2 R_3 i R_1 R_2 i R_1 R_3 = -1 \\ &\neq \nu(XII) \times \nu(IYI) \times \nu(IIY) = (R_2 R_3)(R_1 R_2)(R_1 R_3) = 1 \end{aligned}$$

- Contextuality is eliminated using efficient classical communication $\Rightarrow$ failure of parameter independence.

Reinterpreting the model



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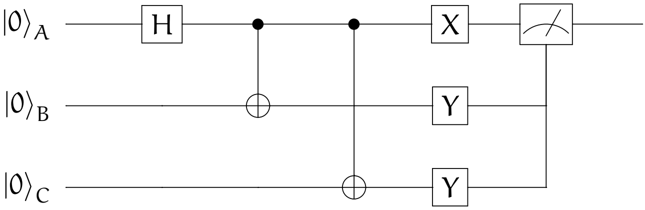

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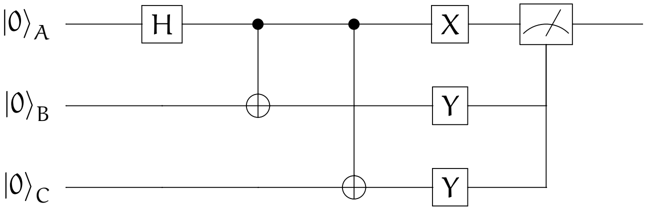

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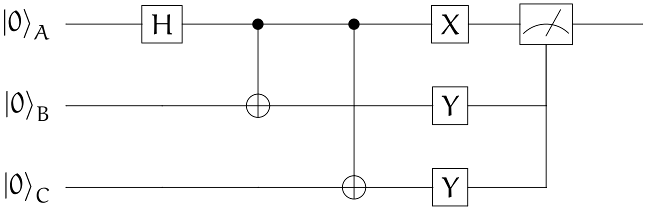

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- Interpreted this way the model is a LHVT.

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Explicating the notion of a local hidden variables theory

What question are we answering with a LHVT?

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  - Most "loopholes" to the Aspect et al. (1981, 1982a,b) experiments do not satisfy these plausibility constraints and are ruled out of consideration on these grounds.
- Purely conceptual question: what is logically possible and still consistent with the predictions of quantum mechanics?
  - Toy theories: no plausibility constraints whatsoever.
  - Useful for making conceptual points; e.g., outcome independence $\neq$ separability (Maudlin, 2011).

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  - Plausibility constraints: must be efficiently Turing computable.
  - We aim to capture the observables, not the beables.

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  - **Yes it is.** A system described by this LHVT will produce the same actually observed (Pauli) measurement statistics as a quantum mechanical one with a number of additional resources that is polynomial (indeed, linear!) in n.
  - I.e. it is no harder for this LHVT than it is for a quantum system.

The GHZ argument

Consider three spatially separated (in the same plane) spin-1/2 systems a, b, c are in the state:

$$|\text{GHZ}\rangle_3 = \frac{1}{\sqrt{2}}(|0\rangle_a|0\rangle_b|0\rangle_c + |1\rangle_a|1\rangle_b|1\rangle_c).$$

Show that a LHVT predicts that

$$v(X^a \otimes X^b \otimes X^c) = v(X^a) \cdot v(X^b) \cdot v(X^c) = -1.$$

Point out that this contradicts quantum mechanical prediction of:

$$v(X^a \otimes X^b \otimes X^c) = 1.$$

The significance of the GHZ argument

This is an altogether more powerful refutation of the existence of elements of reality than the one provided by Bell's theorem for the two-particle EPR experiment. Bell showed that the elements of reality inferred from one group of measurements are incompatible with the statistics produced by a second group of measurements. Such a refutation cannot be accomplished in a single run, but is built up with increasing confidence as the number of runs increases [...] In the GHZ experiment, on the other hand, the elements of reality require a class of outcomes to occur all of the time, while quantum mechanics never allows them to occur. [...] I recently declared in writing that no set of experiments, real or gedanken, was known that could produce such an all-or-nothing demolition of the elements of reality. With a bow of admiration to Greenberger, Horne and Zeilinger, I hereby recant (Mermin, 1990).

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  - Real contradiction arises only for general measurements (i.e., those outside the group of Pauli measurements). These can't be reproduced efficiently.
  - But this contradiction is only statistical, so GHZ's argument is no more powerful than Bell's argument.

Two senses of sufficiency

1. Simply possessing an entangled state is sufficient for speedup.

2. Entanglement is sufficient as a resource to enable speedup

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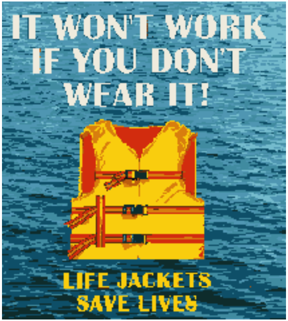

Outline

1. Introduction.
2. The Gottesman-Knill theorem.
3. Analysis of the Gottesman-Knill operations.
   i. The CHSH inequality.
   ii. The GHZ argument.
4. Conclusion.

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