Reduction and emergence: in SSB and in Renormalization

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Introduction

I hold that reduction and emergence are compatible. I take reduction of theories *à la* Nagel: as deduction, usually using judiciously chosen definitions (bridge-laws). And I take emergence as behaviour or properties that are novel.

Work by Batterman, Berry, Kadanoff et al. prompted my morals (2011):
(Deduce): Reduction and emergence are often combined by one theory being deduced as a limit of another, as some parameter $N \to \infty$: the novel behaviour occurs at the limit.
(Before): There is a weaker, yet still vivid, novel and robust behaviour that occurs for finite N—it is this weaker behaviour which is physically real.

I will discuss:

(1): Spontaneous symmetry breaking, following Landsman 1305.4473; (cf. also: Landsman and Reuvers, 1210.2353): Sections 1 and 2.

The idea will be that even for these phenomena, my claim (Before) holds good.

(2): Renormalization (Sections 3 to 6). The idea will be that the modern approach to renormalization illustrates Nagelian reduction, and my morals.

Outline

The challenge of SSB—and Schrödinger's cat

1. One has a fundamental theory formulated in terms of a parameter $N < \infty$ (quantum statistical mechanics) or $\hbar > 0$ (quantum mechanics);
2. The ground state (or equilibrium state) of some Hamiltonian with symmetry group G is unique and hence G-invariant for any value of $N < \infty$ or $\hbar > 0$;
3. The $N \to \infty$ or $\hbar \to 0$ limit of the ground state (etc.) exists, is still G-invariant, but is now mixed (non-extremal);
4. The limit theories at $N = \infty$ (infinite-volume quantum statistical mechanics) or $\hbar = 0$ (classical mechanics) exist on their own terms (i.e. without taking any limit) and are completely understood;

1. These limit theories may display SSB (depending on the model): they may have a family of G-variant pure ground states (extremal KMS states), forming a G-space;
2. Nature may display SSB, in which case physical samples modelled by such Hamiltonians behave like the limit theory (although in reality $N < \infty$ or $\hbar > 0$);
3. Thus for any $N < \infty$ or $\hbar > 0$, the fundamental theory neither approximates the limit theory nor models reality correctly: indeed, it spectacularly and totally fails to do so.

The solution: perturbed ground states converge to the desired classical state

The key to get rid of these Schrödinger Cat states is to broaden one's view and also take the first excited state into account.

As the parameter goes to its limit, the energy difference between the ground and first excited state goes to zero fast enough that almost any asymmetric perturbation will make the ground state of the perturbed Hamiltonian be asymmetric (i.e. symmetry-breaking) *at finite parameter-values*, in the appropriate way.

So: (i) for the quantum Ising chain, spins up *or* down;
(ii) for the double-well potential, left well *or* right well.

So the behaviour of real samples is well approximated by the theory at very large N or very small $\hbar$.

The reason lies in the exponential instability of the ground state under asymmetric perturbations for large N or small $\hbar$, which should cause the system to pick a specific symmetry-breaking ground state already for some *finite* value of N or *positive* value of $\hbar$ (as opposed to the limiting values $N = \infty$ or $\hbar = 0$, which are physically irrelevant).

So (Before) is vindicated, even for the challenging case of symmetry-breaking.

Plan and motivation

I will now argue that the explanation, using renormalization group ideas, of why non-renormalizable terms dwindle at long distances amounts to a family of Nagelian reductions.

For a renormalization scheme that defines a flow to lower energies amounts to a set of definitions that enable deductions, from a theory describing high-energy physics, of a low-energy theory. Because the same scheme shows how many similar high-energy theories flow to correspondingly similar low-energy theories, we have a unified family of reductions. And taking renormalizability as the emergent behaviour, (Deduce) and (Before) also hold.

This undermines the impression (Batterman, Cao and Schweber, Bain) that the modern approach to renormalization, and the idea of effective field theories, yields: (i) anti-Nagelian morals, and-or (ii) anti-reductionist morals.

Nagel endorsed

Take theories to be sets of sentences closed under deducibility (no doubt, unformalized). T_t (t for 'top') is reduced to T_b (b for 'bottom') by:
(1) being logically deducible from T_b, usually together with judiciously chosen definitions (bridge laws); and
(2) the deduction satisfying informal constraints, like each *definiens* playing a role in T_b.

The constraints in (2) answer the charge that deducibility is too weak. Nagel answered the charge that it is too strong, by allowing *approximative reduction*: there need only be a strong analogy between T_t and what strictly follows from T_b.

We can already see how this fits with my morals, (Deduce) and (Before). T_t corresponds to an $N = \infty$ limit of T_b. In some cases, it is not really true. But (Before): the weaker form of novel behaviour occurs at finite N, and strictly follows from T_b.

Heuristic but successful theories

QED and the other interacting QFTs are not rigorously defined.

So we adopt a perturbative approach, with power series $\Sigma\, g^n A_n$. There are various limitations: especially, the A_n are typically infinite.

And yet it works! E.g. we predict the Lamb shift up to some ten significant figures. This is like predicting the measured diameter of the USA to within the width of a human hair.

Simplifications at higher energies

Some of our current theories are increasingly *well-behaved* at higher energies. In QCD, the strong force gets weaker at higher energies. So QCD is *asymptotically free*.

So it is reasonable to hope that it will be rigorously defined.

On the other hand, QED gets worse: so as the archetypal example of a QFT, it gives a pessimistic impression.

The need for corrections

Classically, there is no problem about using the measured force F felt by a test-particle at a given distance r from the source, to calculate the charge (*coupling constant*) of the source particle.

Example: electrostatics: $F = -e/r^2$. So we invert this equation to calculate that the source's charge is: $e = -F.r^2$.

In QFT, there are corrections which depend on the energy μ at which we describe or probe the system.

We write $g(\mu)$ for the *physical coupling constant*, measured at energy μ.
$g(\mu)$ is a function of g_0, the *bare coupling constant* in the fundamental equations: $g(\mu) = g(\mu, g_0)$.
Inverting the equation: $g_0 = g_0(g(\mu))$.

The cut-off introduced

Typically, A_n is an integral over momenta k, including arbitrarily high momenta. A_n looks like $\int_{k_0}^{\infty} dk\ k^a$, with $a \geq -1$, or even $a > 0$, so that the integral is infinite.
To get a finite answer, we impose a *cut-off* Λ, and so use $\int_{k_0}^{\Lambda} dk\ k^a$.

In quantum theory, high momenta correspond to short distances. So the cut-off Λ corresponds to ignoring field contributions that vary on a distance shorter than a distance d. So: $g(\mu) = g(\mu, g_0, d)$.

Similarly, μ corresponds to a length L: $\mu \sim 1/L$. So we can write: $g(L) = g(L, g_0, d)$.

The task is to measure $g(L)$ and then invert the equation to write: $g_0 = g_0(g(L), d) = g_0(L, d)$.

Letting the cut-off d go to zero

But d is up to us. So if:
(a) spacetime is continuum, and
(b) our theory holds good at arbitrarily short lengths,
then surely g_0 goes to a limit (at least at some, e.g. the observed, μ or L):

$$\text{with } g(\mu) \text{ at observed value: there exists } \lim_{d \to 0} g_0 \,. \tag{1}$$

If this limit is finite, i.e. $\in \mathbb{R}$, we say: *the theory is finite*.
Beware: this does not mean the power series $\Sigma g^n A_n$ converges.

QED is not finite. As $d \to 0$, the bare electric charge goes to infinity.

The consensus is that this is *acceptable*. So if the limit exists, even if it $= \pm\infty$, we say the theory is *renormalizable*.

Dyson's criterion

Express all dimensions in terms of length. Let the bare coupling constant have dimensions of length^D.

Then: the theory is *renormalizable* iff $D \leq 0$. For:

$$g(L)/g_0 \sim (L/d)^{-D} \tag{2}$$

Thus if $D > 0$, the exponent on the right-hand side will be negative. So when L is much smaller than d, $L << d$, the right hand side is very large. So the interaction gets stronger at high energies.

Happily, our best theories, QED, QFD and QCD, are all renormalizable. So the traditional approach says: what *good fortune*!

The modern approach to renormalization

A "vector" of coupling constants $(g_1(\mu), ..., g_N(\mu))$ represents a point in an N-dimensional space. As μ varies, this point flows through the space. The *renormalization group flow.* For us, the main idea is:

Good fortune explained: non-renormalizable terms dwindle at longer distances:
Recall

$$g(L)/g_0 \sim (L/d)^{-D} \tag{3}$$

Thus if $D > 0$ (i.e. non-renormalizable), and L is much larger than the cut-off d, then the right hand side of eq. 3 is very small.

So at large distances/low energies, the non-renormalizable contributions dwindle. The high-energy behaviour "decouples" from the low-energy behaviour.

Notice that this explanation does not depend on spacetime being a continuum. Provided the theory is true enough at some high, maybe inaccessible, energies, so as to validate eq. 3, then: at much lower energies, we see only renormalizable interactions.

Renormalizability deduced at low energies as a family of Nagelian reductions

Let T_b describe high-energy physics, maybe with non-renormalizable terms.

A renormalization scheme that defines a flow to lower energies amounts to a set of definitions, i.e. bridge-laws:
that enable a deduction of a T_t which describes low-energy physics, with negligible non-renormalizable contributions.

Because the non-renormalizable contributions are "merely" negligible, this is approximative reduction.

Because the same scheme shows how many similar theories T_b flow to correspondingly similar theories T_t, we have a unified family of Nagelian reductions.

The morals (Deduce) and (Before) are again illustrated:—

(Deduce): The parameter N is length L (or inverse energy). The infra-red fixed point is the limiting ($N = \infty$) theory T_{t*}. The novel behaviour is renormalizability.

(Before): at $N \equiv L$ large enough, the non-renormalizable terms are negligible (approximative reduction).

(Cf. Butterfield and Isham (2000, p. 79), Hartmann (2001, p. 295), Castellani (2002, p. 263).)

Philosophical remarks

(1): *Accepting theories*:
Treating interacting quantum field theories as well enough defined to be theories in Nagel's sense goes against some authors, e.g. Kaiser, who say that (i) QED, and other such *so-called* interacting QFTs, do not deserve the name 'theory', and so (ii) the notion of theory is not useful for the analysis, historical or philosophical, of this part of physics.

But we should distinguish whether 'theory' means 'theory in general' or 'a specific theory'. So one can doubt or deny that theory in general is a useful notion, while maintaining that a specific theory is. Cf. Hartmann (2001, 291-297).

(2): *No meaning variance*:
In these reductions, there is no threat of meaning variance. As the energy-scale changes, the coupling constants etc. change. But there is no radical change of meanings or of concepts, and there is always 'strong analogy'.

(3): *Multiple realizability*:
Under a single renormalization scheme, two very dissimilar theories $T_b, T_{b'}$ can flow to two very similar theories $T_t, T_{t'}$, or even to a single one T_{t^*}. We can describe this as either:
(i): (individuating reductions by a trajectory of the flow): two local reductions, one by T_b, and one by $T_{b'}$; or
(ii): (individuating reductions by the renormalization scheme used): a single reduction.
Either way, we have multiple realizability, in philosophers' sense.

(4): *Dynamical systems*: Recall the idea of 'dynamical systems reduction': viz, time-evolution commutes with the coarse-graining/projection of a state-space ... upto some errors, and on a short enough time-scale. E.g. Ehrenfest's theorem, Lanford's theorem. Does this apply here?

No: because scattering theory abstracts from time-evolution except in the asymptotic sense; and one cannot expect the evolution on smaller time-scales to commute with the transition $T_b \to T_t$.

But agreed: the transition $T_b \to T_t$ is like a coarse-graining of the state-space, since it corresponds to integrating out high-k degrees of freedom.

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