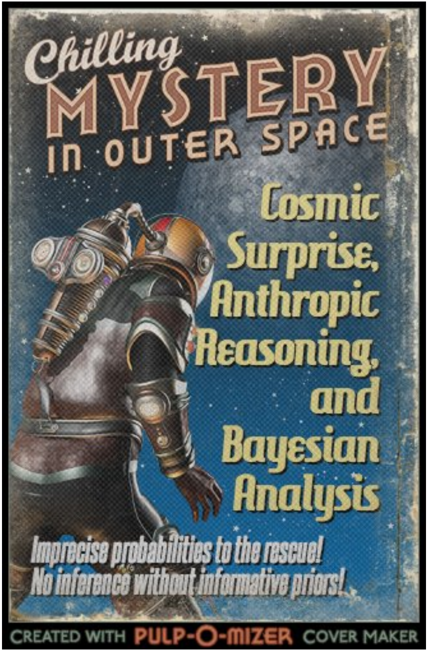
Chilling
MYSTERY
IN OUTER SPACE
Cosmic Surprise, Anthropic Reasoning, and Bayesian Analysis
Imprecise probabilities to the rescue!
No inference without informative priors!
CREATED WITH PULP-O-MIZER COVER MAKER

Cosmic Surprise, Anthropic Reasoning, & Bayesian Analysis

Foundations of Physics
MCMP–LMU, July 30, 2013

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ENGAGING SCIENCE.

Abstract

There are, in cosmology, improper applications of the principle of indifference in Bayesian analysis. Imprecise credences constitute a Bayesian-friendly framework that allows us to avoid inadequate neutral priors, and better handle ignorance.

Outline

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- A Bayesian notion of neutral support
- What role for anthropic reasoning?

Cosmic surprises

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Cosmic surprises

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  - Multiverse?
- Dirac's coincidence (1937):

$$\frac{\text{radius universe}}{\text{radius electron}} \simeq \frac{F_{Ep-e}}{F_{Gp-e}}$$

Bayesian confirmation for cosmic surprises

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> *. . . following Laplace's principle of indifference, we often start with the least informative probability distribution, i.e., a flat distribution, and then sharpen it in the light of additional information. This is, as we see it, the essence of the Bayesian approach. [Gibbons and Turok(2008)]*

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3. Propose alternative theory: $p(\alpha|T') \simeq 1$, $p(\alpha|T_{BB}) << 1$.

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3. Propose alternative theory: $p(\alpha|T') \simeq 1$, $p(\alpha|T_{BB}) << 1$.
4. Similar priors: $p(T_{BB}) \simeq p(T')$
   $\rightarrow T'$ confirmed: $p(T'|\alpha) >> p(T_{BB}|\alpha)$.

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- Low probability $\equiv$ ignorance $\to$ explanation $\equiv$ confirmation. Confirmation whenever any coincidence "explained".
- Assumes able to assign $p(\text{coincidence})$.

- Source of problems: adequacy of priors, POI, or...
- ...Bayesian analysis altogether?

Bayesian reasoning

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- Belief comes in degrees,

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- which obey the laws of probability:
  - $p \geq 0$
  - $p(\top) = 1$,
  - additivity: a, b incompatible $\rightarrow p(a \& b) = p(a) + p(b)$.

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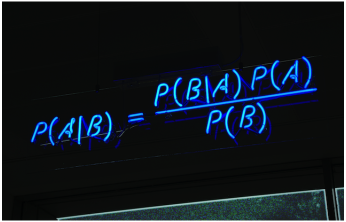
- Learning: updating a prior $\rightarrow$ posterior.



Bayesian reasoning

- Belief comes in degrees,
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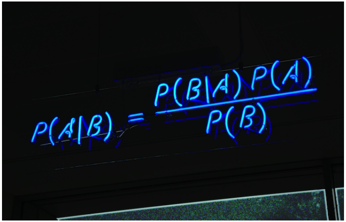
- Learning: updating a prior $\rightarrow$ posterior.
- Rational agents choose highest expected value.



Confirming-not vs. not-confirming

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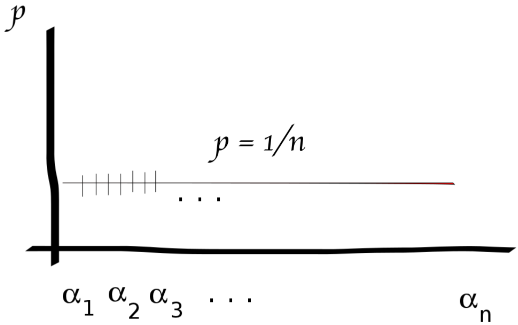

POI + additivity

$$p(k \cdot \alpha_i) = \frac{k}{n} \geq \frac{1}{n}$$

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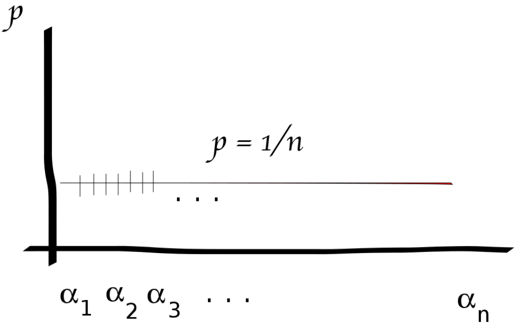

POI + additivity

$$p(k \cdot \alpha_i) = \frac{k}{n} \geq \frac{1}{n}$$

- $\neq$ ignorance!
- no clue about α_i $\neq$ α_i improbable

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- Sharpness not necessary [Levi(1974), Walley(1991), Joyce(2010)].
- Range problematic [Elga(2010)]

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- C can be sharp, or not.
- C not nec. additive.
- Comparative probabilities even if imprecise.

Neutral support

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Distinction between:

- stochastic independence: $C(\alpha_i|\alpha_j) \equiv C(\alpha_i|\neg\alpha_j)$,
- unknown interaction.

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- and $C(\top) = 1 - C(\bot) = 1$.

Norton's criteria for neutral support

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- invariant under negation: $C(\alpha) \equiv C(\neg\alpha)$.

Norton's objections to imprecise credences

(. . .) ignorance is not the maintaining of all possible beliefs at once; it is the maintaining of none of them. [Norton(2007a)]

The sort of ignorance I seek to characterize is first order ignorance; it is just not knowing which is the true outcome (. . .). [Norton(2007b)]

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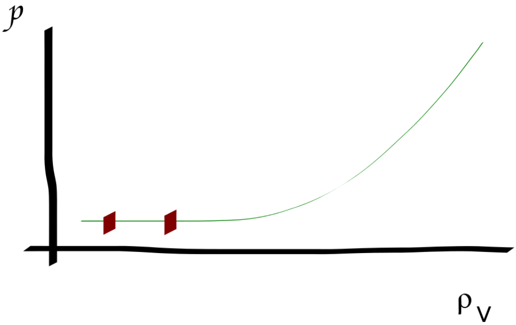
- ρ_V unexplained,
- $\dfrac{\text{part. phys.}}{\text{obs}} = 10^{120}$.
- Anthropic bounds on $\rho_V \rightarrow$ prediction?
    - ρ_V too large: no structure formation,
    - ρ_V too small: universe recollapses too early.

Can anthropic considerations explain the value of the cosmological constant?



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- Assume typicality: prediction around mean value within anthropic range.

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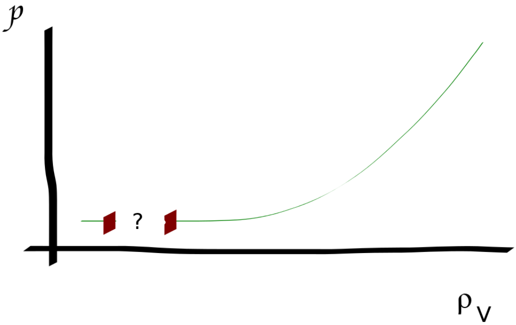

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Conclusion

Imprecise model provides better representation of ignorance, prevents uninformative priors from doing too much inductive work.

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Thank you!

Danke sehr!

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